

How safe is nuclear power? A statistical survey suggests less than expected

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Appendix: Statistical Analysis

Trevor Sweeting, UCL Department of Statistical Science, t.sweeting@ucl.ac.uk

Thomas Rose, Münster University of Applied Sciences, Germany, and UCL Department of
Science and Technology Studies, thomas.rose@ucl.ac.uk

1 Statistical model

The notation below is common to the analyses of The Guardian data(Rogers 2011b) and the Sovacool data (Sovacool 2008, 2010, 2011).

Let n_t be the number of reactors operational in year t , coded as $t = 1, \dots, T$ and let Y_{tr} be the number of accidents at reactor r , for $r = 1, \dots, n_t$, in year t . We assume that accidents at a given reactor in any given year occur independently. Then accidents at that reactor over a one year period will occur according to a (possibly nonhomogenous) Poisson process, so that Y_{tr} will be distributed as $\text{Poisson}(\lambda_{tr})$, where λ_{tr} is the expected number of accidents at reactor r in year t , or approximately the probability of at least one accident at the reactor in year t . Further assuming independence of the Y_{tr} over the reactors operational at time t , it follows that the total number of accidents $Y_t = \sum_{r=1}^{n_t} Y_{tr} \sim \text{Poisson}(\lambda_t)$, where $\lambda_t = \sum_{r=1}^{n_t} \lambda_{tr}$ is the expected total number of accidents in year t .

We will assume for simplicity that $\lambda_{tr} = e_t$, a constant, so that $\lambda_t = n_t e_t$ and e_t is the expected number of accidents per reactor per year. Although this assumption is unlikely to be true, a small amount of variation across reactors will not unduly affect the results obtained here. Any such variation will lead to extra-Poisson variation, which can be assessed following model fitting.

Let $N_t = \sum_{u=1}^t n_u$ be the cumulative number of reactor-years at year t . We use N_t as a

measure of operational experience in year t and postulate that e_t is a function of N_t , so that

$e_t = e(N_t)$. Without any loss of generality we can write $e(N) = \alpha \exp\left\{-\int_0^N \beta(x)dx\right\}$, where $\beta(N)$

is the (instantaneous) rate of learning when the number of reactor-years has reached N .

Let $X_t = \sum_{u=1}^t Y_u$ be the cumulative number of accidents up to time t . Assuming independence of

the Y_t 's, we have $X_t \sim \text{Poisson}(\Lambda_t)$, where $\Lambda_t = \sum_{u=1}^t \lambda_u = \sum_{u=1}^t n_u e(N_u)$. If there is no learning then

$\beta = 0$ and $e(N_t) = \alpha$, which is the constant expected number of accidents per reactor per year,

from which it follows that the expected cumulative failure rate, $E\left(\frac{X_t}{N_t}\right) = \alpha$, a constant. If,

however, there is learning then $\beta > 0$ and $e(N)$ will be a decreasing function of N so that a plot of

$\frac{X_t}{N_t}$ against N_t will exhibit a decreasing trend. The Poisson distribution can be used to set

pointwise confidence limits on $E\left(\frac{X_t}{N_t}\right)$. For simplicity, approximate 2- σ bounds have been

incorporated into Fig 2 and 3 for $\log E\left(\frac{X_t}{N_t}\right)$.

For The Guardian data, we take $\beta(N) = \beta$, so that the rate of learning is constant, and $e(N) = \alpha \exp(-\beta N)$, an exponentially decreasing function of the number of reactor-years. Since $\log \lambda_t = \log n_t + \log \alpha - \beta N_t$, the model is a generalised linear model (McCullagh & Nelder 1999) with Poisson family and log link function. The analysis was implemented in the programming language R.

In the case of the Sovacool data, we use the biexponential function given by

$$e(N) = \alpha_0 e^{-\beta_0 N} + \alpha e^{-\beta N}.$$

A convenient parameterisation of this function is

$$e(N) = \alpha e^{-\beta N} \{1 + e^{-\eta(N-\phi)}\}$$

where $\eta = \beta_0 - \beta$ and $\phi = \frac{\left\{ \log\left(\frac{\alpha_0}{\alpha}\right) \right\}}{\eta}$. In this parameterisation the instantaneous learning

rate is

$$\beta(N) = \beta + \frac{\eta}{1 + e^{\eta(N-\phi)}}$$

In particular, the initial rate is $\beta_I = \beta + \frac{\eta}{1 + e^{-\eta\phi}}$ and the final rate is simply β . If the

change from the initial to the final rate is quite pronounced then it can be shown that this model will also approximate to a change-point model, with the change-point at $N = \phi$.

We can now set up the likelihood function $L(\theta)$, where $\theta = (\gamma, \beta, \phi, \eta)$ and $\gamma = \log \alpha$, and carry out a likelihood analysis (Garthwaite, Jolliffe & Jones 2006). Starting values for the computation can be obtained from graphical inspection and/or by fitting a generalised linear model to the data after 1962, using the Poisson family with a log link function. The maximum likelihood estimates of the parameters can then be computed, along with their approximate standard errors, and appropriate likelihood ratio tests carried out. The approximate confidence interval for the change-point ϕ was obtained from the profile likelihood of $\log(\phi)$.

The results are fairly insensitive to the choice of alternative ranges of years. As a

diagnostic for the model we calculated the standardised response residuals $r_t = \frac{y_t - \hat{\lambda}_t}{\sqrt{\hat{\lambda}_t}}$ from the

observed values y_t of Y_t and the estimated model values $\hat{\lambda}_t$. When plotted against year these

show no particular unusual pattern. Moreover, the observed standard deviation of these residuals is 0.982, indicating that our initial assumption that λ_{tr} is constant over reactors is a reasonable one. Specifically, if we suppose that there is a positive but constant variation over reactors, so that $\text{var}(\lambda_{tr}) = \sigma^2$, then the theoretical variance of the t^{th} residual at the true parameter values will be $1 + e(N_t)\sigma^2$. Thus the observed residuals would exhibit extra-Poisson variability, which does not appear to be the case here.

In view of the sharp change between the initial and final learning regimes, the data could alternatively be modelled by a change-point process with

$$\beta(N) = \begin{cases} \beta_I, & N \leq \phi \\ \beta, & N > \phi \end{cases}.$$

This model produces very similar results. However, we prefer the biexponential modelling as it does not presuppose the existence of a sudden change in the failure rate.

2 Appendix The Guardian list

The following list of nuclear accidents has been compiled by the Guardian(Rogers 2011a)

Year	Site	INES	Country	Description
2011	Fukushima	5	Japan	Reactor shutdown after the 2011 Sendai earthquake and tsunami
2011	Onagawa		Japan	Reactor shutdown after the 2011 Sendai earthquake and tsunami caused a fire
2006	Fleurus	4	Belgium	Severe health effects for a worker at a commercial irradiation facility as a result of high doses of radiation
2006	Forsmark	2	Sweden	Degraded safety functions for common cause failure in the emergency power supply system at nuclear power plant
2006	Erwin		US	Thirty-five litres of a highly enriched uranium solution leaked during transfer
2005	Sellafield	3	UK	Release of large quantity of radioactive material, contained within the installation
2005	Atucha	2	Argentina	Overexposure of a worker at a power reactor exceeding the annual limit
2005	Braidwood		US	Nuclear material leak
2003	Paks	3	Hungary	Partially spent fuel rods undergoing cleaning in a tank of heavy water ruptured and spilled

				fuel pellets
1999	Tokaimura	4	Japan	Fatal overexposures of workers following a criticality event at a nuclear facility
1999	Yanangio	3	Peru	Incident with radiography source resulting in severe radiation burns
1999	Ikitelli	3	Turkey	Loss of a highly radioactive Co-60 source
1999	Ishikawa	2	Japan	Control rod malfunction
1993	Tomsk	4	Russia	Pressure buildup led to an explosive mechanical failure
1993	Cadarache	2	France	Spread of contamination to an area not expected by design
1989	Vandellos	3	Spain	Near accident caused by fire resulting in loss of safety systems at the nuclear power station
1989	Greifswald		Germany	Excessive heating which damaged ten fuel rods
1986	Chernobyl	7	Ukraine (USSR)	Widespread health and environmental effects. External release of a significant fraction of reactor core inventory to the environment from explosion of a high activity waste tank."
1986	Hamm-Uentrop		Germany	Spherical fuel pebble became locked in the pipe used to deliver fuel elements to the reactor
1981	Tsuraga	2	Japan	More than 100 workers were exposed to

				doses of up to 155 millirem per day radiation
1980	Saint Laurent des Eaux	4	France	Melting of one channel of fuel in the reactor with no release outside the site
1979	Three Mile Island	5	US	Severe damage to the reactor core
1977	Jaslovske Bohunice	4	Czechoslovakia	Damaged fuel integrity, extensive corrosion damage of fuel cladding and release of radioactivity
1969	Lucens		Switzerland	Total loss of coolant led to a power excursion and explosion of experimental reactor
1967	Chapelcross		UK	Graphite debris partially blocked a fuel channel causing a fuel element to melt and catch fire
1966	Monroe		US	Sodium cooling system malfunction
1964	Charlestown		US	Error by a worker at a United Nuclear Corporation fuel facility led to an accidental criticality
1959	Santa Susana Field Laboratory		US	Partial core meltdown
1958	Chalk River		Canada	Due to inadequate cooling a damaged uranium fuel rod caught fire and was torn in two

1958	Vinca		Yugoslavia	During a subcritical counting experiment a power buildup went undetected - six scientists received high doses
1957	Kyshtym	6	Russia	Significant release of radioactive material
1957	Windscale Pile	5	UK	Release of radioactive material to the environment following a fire in a reactor core
1952	Chalk River	5	Canada	A reactor shutoff rod failure, combined with several operator errors, led to a major power excursion of more than double the reactor's rated output at AECL's NRX reactor

3 Figure Captions

Fig 1 Number of power reactors worldwide

Fig 2 Cumulative probability = Cumulative accidents / cumulative reactor years in a log scale vs cumulative reactor years, each data point representing one year, the lines represent the 95% confidence limits, source: The Guardian

Fig 3 Cumulative probability = Cumulative accidents / cumulative reactor years in a log scale vs cumulative reactor years, each data point representing one year, the lines represent the 95% confidence interval, source: Sovacool

Fig 4 Observed and theoretical annual accident rates

Fig 5 Same as Fig 4 with different y-scale

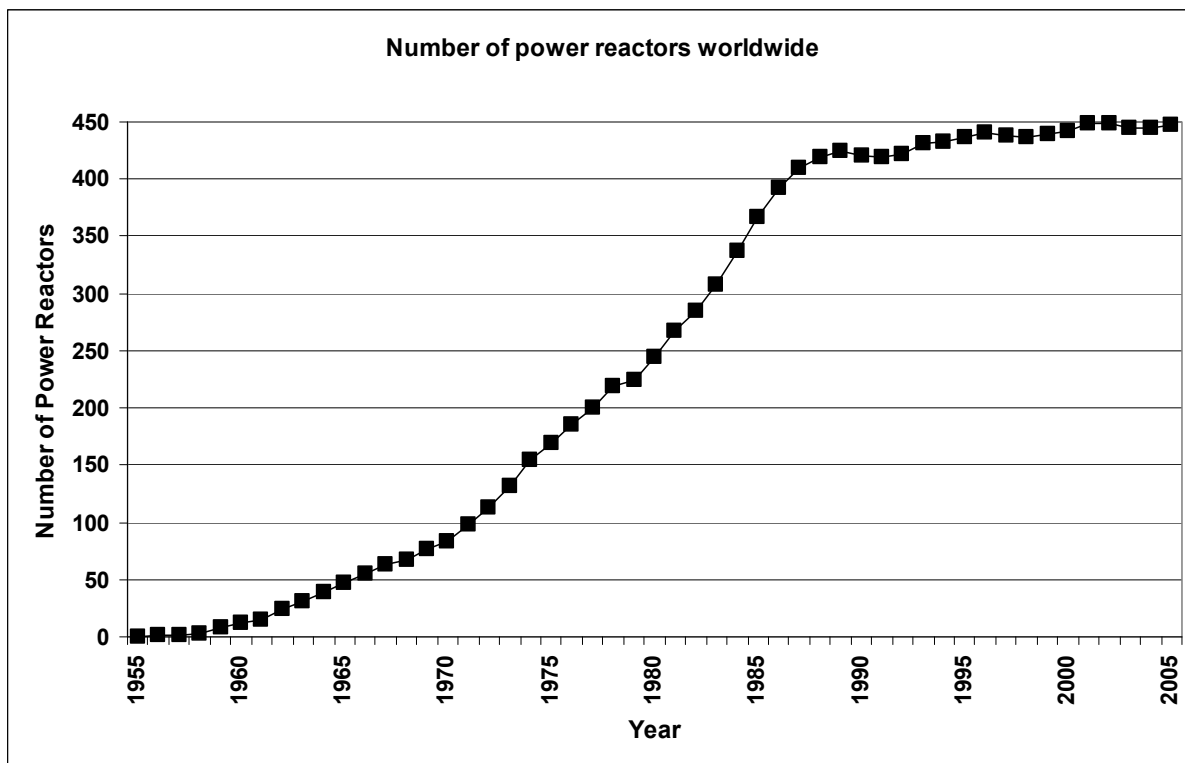


Fig 1 Number of power reactors worldwide

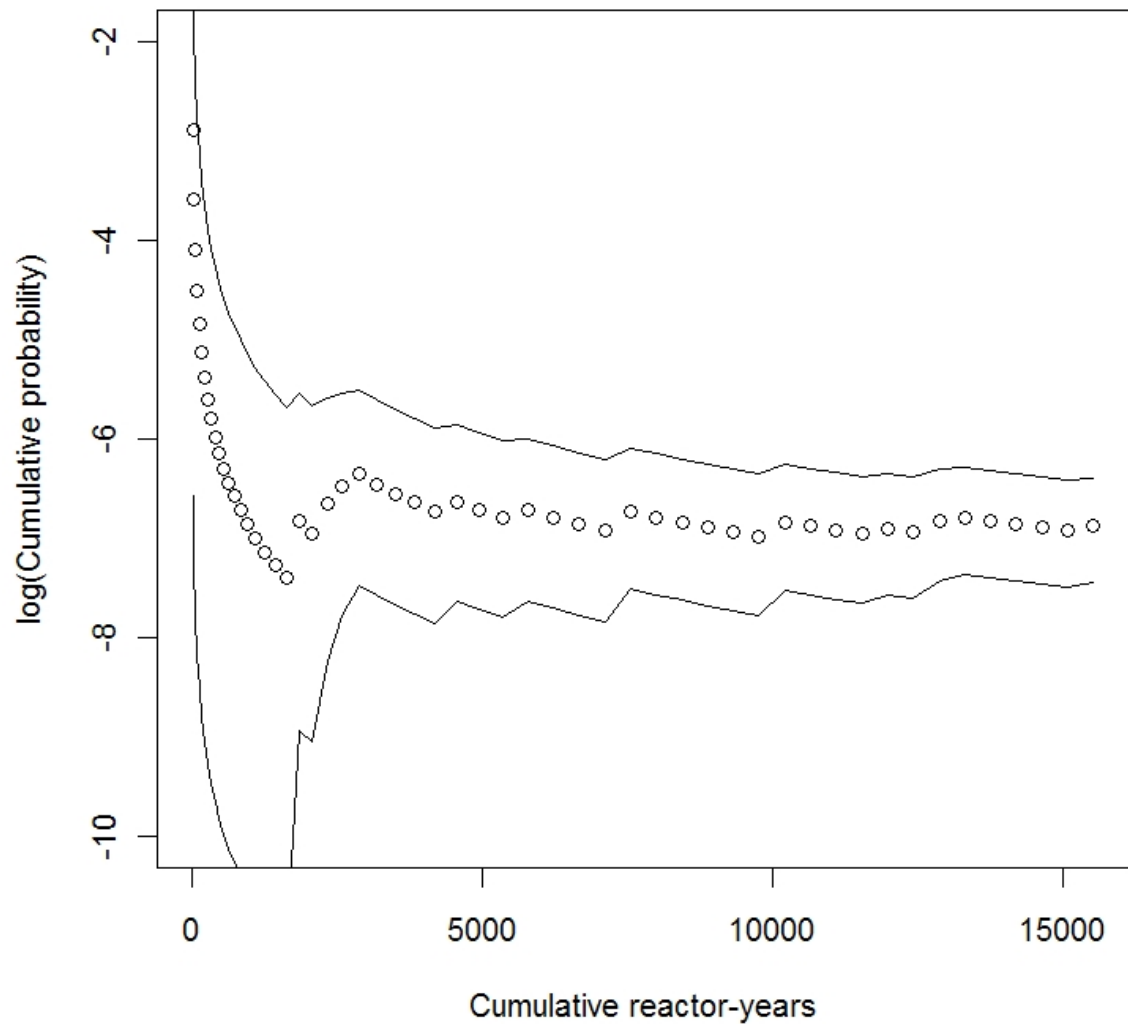


Fig 2 Cumulative probability = Cumulative accidents / cumulative reactor years in a log scale vs. cumulative reactor years, each data point representing one year, the lines represent the 95% confidence limits, source: The Guardian

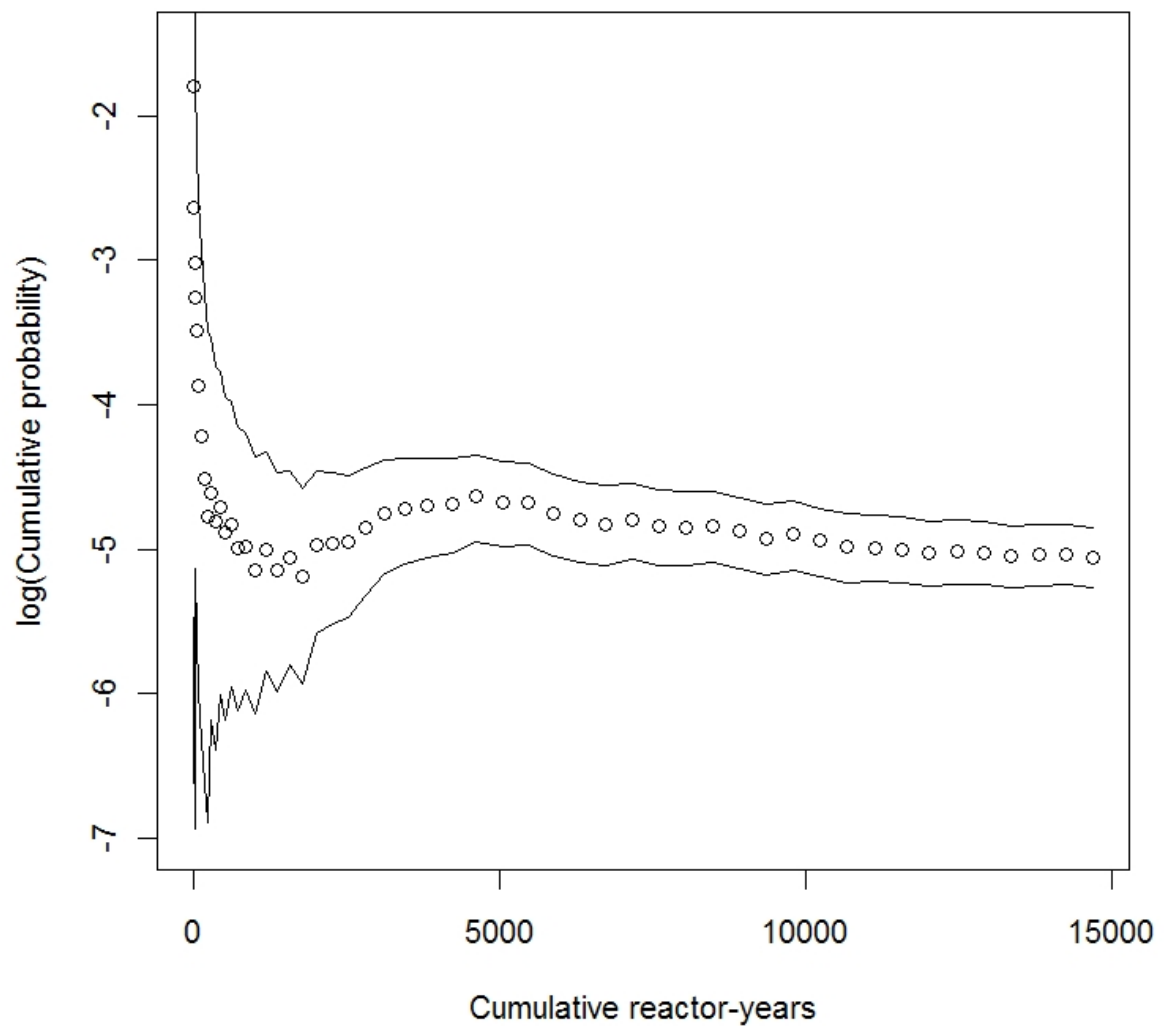


Fig 3 Cumulative probability = Cumulative accidents / cumulative reactor years in a log scale vs. cumulative reactor years, each data point representing one year, the lines represent the 95% confidence interval, source: Sovacool

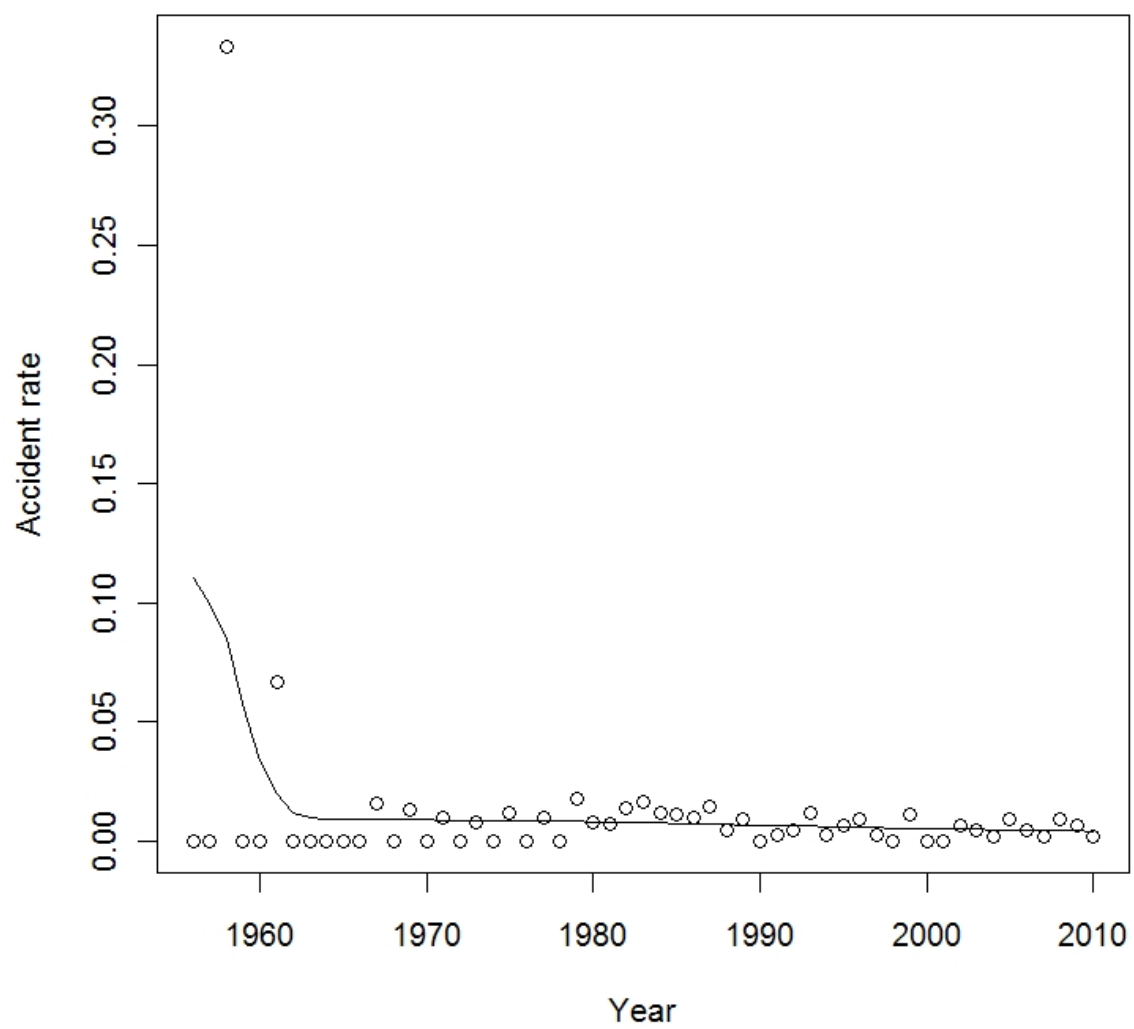


Fig 4 : Observed and theoretical annual accident rates

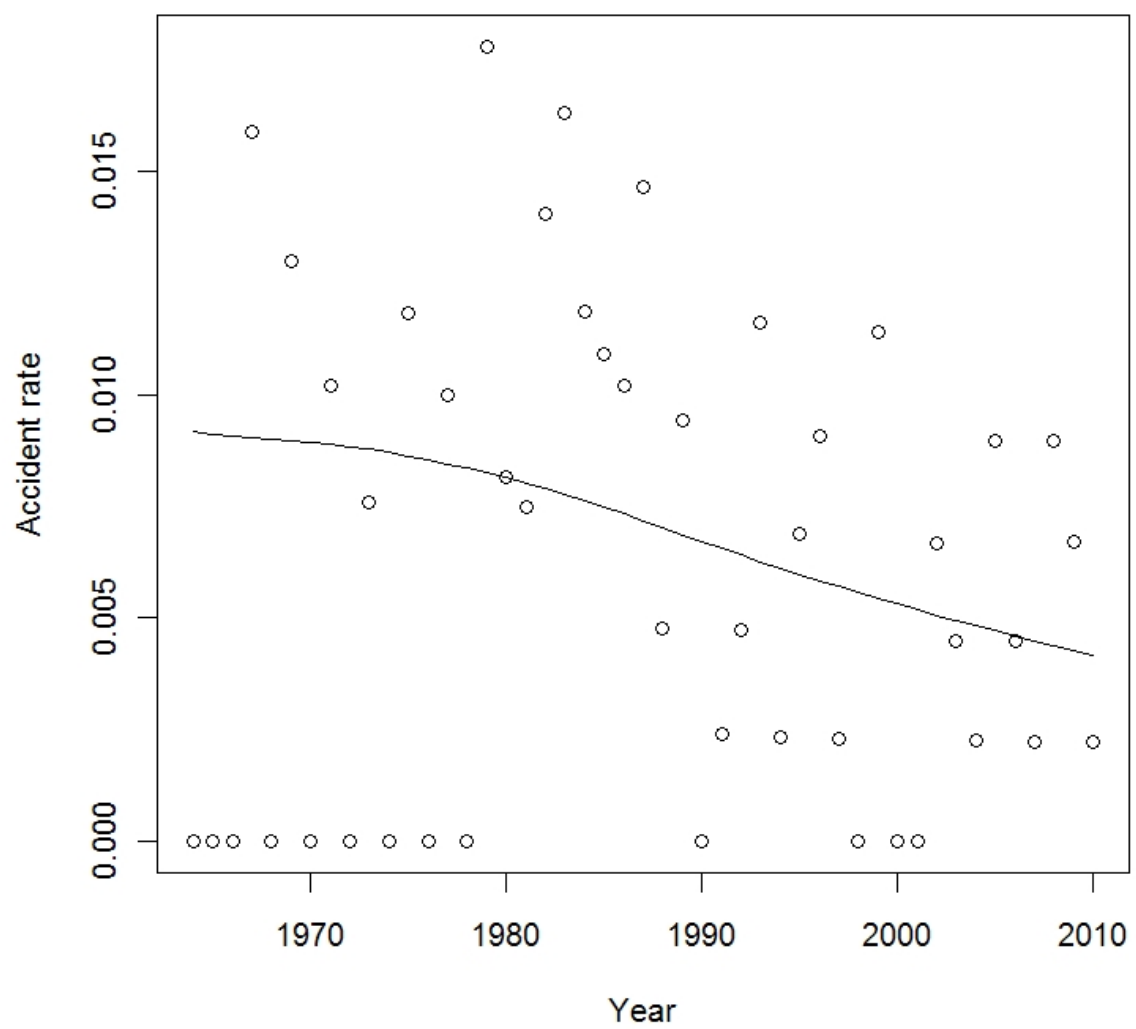


Fig 5 Same as Fig 4 with different y scale

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